

Exam - PoMS, 23/01/2014

- Write each question on a sheet of paper.
- Write your **name** and **student ID** on each sheet.
- Pay attention to units. A numerical result without a unit will be considered wrong!
- Only a regular calculator is allowed.
- This is **NOT** an open book exam.
- You are allowed to bring one A4 page with your own notes (one side only).
- You have **3 hours** to complete the exam.
- Note: $\mathcal{L}(t^n e^{-at}) = \frac{n!}{(s+a)^{n+1}}$.

Question 1: General (2 points)

- What is a *Schmitt trigger* and what is its application? Describe briefly its working principle.
- Explain the working principle behind a *thermocouple*.
- The *autocorrelation function* is often used to detect the presence of a period signal buried in random noise. Explain qualitatively what the autocorrelation function embeds and how it can be used to identify the presence of a buried signal.
- Describe the working principle of a *tachogenerator* in connection to the concept of *reluctance*.

Question 2: A pressure gauge (2 points)

The table below characterises a pressure gauge designed for operation at room temperature (25 °C, standard condition).

Pressure (bar)	1	2	3	4	5	6
I_{out} [mA] (25 °C)	3.9	7.0	10.1	13.2	16.3	19.4
I_{out} [mA] (35 °C)	3.5	7.2	10.9	14.6	18.3	22.0

- Explain whether the environment variable is modifying, interfering, or both modifying and interfering.
- Determine the values of K_M , K_I , a , and K associated with the generalized model equation $O = (K + K_M \cdot I_M) \cdot I + a + K_I \cdot I_I$. Note down the units of the parameters!

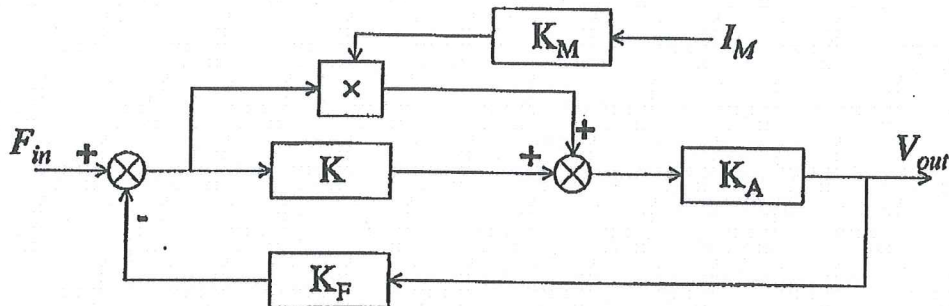


Figure 1: A high-gain negative feedback system.

Question 3: A negative feedback system (2 points)

Figure 1 shows the block diagram of a force transducer (force F_{in} to voltage V_{out}) with a high-gain negative feedback. A sensing element has a sensitivity of K , followed by an amplifier with a high gain K_A , and a feedback element with a sensitivity K_F . The sensing element is influenced by a modifying interference, I_M , with a sensitivity of K_M .

- Derive the exact equation that describes the static behavior of the system.
- The high-gain negative feedback system is designed such that the amplifier gain, K_A , is "large". Give an expression that quantifies the meaning of "large", such that $V_{out} \approx F_{in}/K_F$.
- What is/are the advantage(s) of this technique of "high-gain negative feedback"?

Question 4: A temperature measurement system (2 points)

A temperature measurement system consists of a thermocouple, an amplifier, and a recorder. The thermocouple can be represented by a 1st-order low-pass system with a time constant $\tau = 10$ s and a steady-state sensitivity of 10^{-4} V/°C. The amplifier has a multiplication factor of 10^3 and can be considered as a purely static system. The dynamic response of the total measurement system was found to be $G(s) = 1/(1 + 12s + 20s^2)$, where $G(s)$ is the transfer function with s as the Laplace variable (with units sec^{-1}). Note that the steady-state sensitivity of the complete system is unity.

- Give an expression for the transfer function of the recorder and its steady-state sensitivity.
- The true temperature changes suddenly by 10 °C from a steady-state condition. Find an expression of the change of the temperature given by the recorder.
- Estimate the bandwidth of the total measurement system and motivate your answer.

Question 5: Two-element resistance sensor bridge (2+1 points)

Consider a two-element resistance sensor bridge as indicated in Fig. 2. The bridge consists of two identical metal resistance sensors. One sensor is placed at a temperature T_1 (in °C) and the other placed at a fixed reference temperature $T_2=0$ °C. The formula for resistance as a function of temperature is given in the figure with the temperature coefficient $\alpha=5 \times 10^{-3}$ °C⁻¹ and $R_0=100$ Ω. The sensor indicated with T_1 operates in a range between 0 and 50 °C.

- What is the choice for R_3/R_4 such that $E_{Th}=0$ when $T_1=T_2$ (balanced bridge). Motivate your answer.
- Take $R_3=R_4=R_0=100$ Ω. The system is calibrated by varying the supply voltage V_S such that $E_{Th}=1$ V at $T_1=50$ °C. What value of V_S is required and how large is the non-linearity at $T_1=25$ °C?
- What choice for R_3 and R_4 is needed to improve significantly the linearity of the system and to obtain the relation

$$E_{Th} = V_S \left(\frac{R_0}{R_3} \right) \alpha T_1.$$

What price one pays for achieving linearity? Motivate your answers.

- Take $V_S=12$ V and $R_3=R_4=10$ kΩ. How large is the maximum power dissipation through sensor R_1 ?

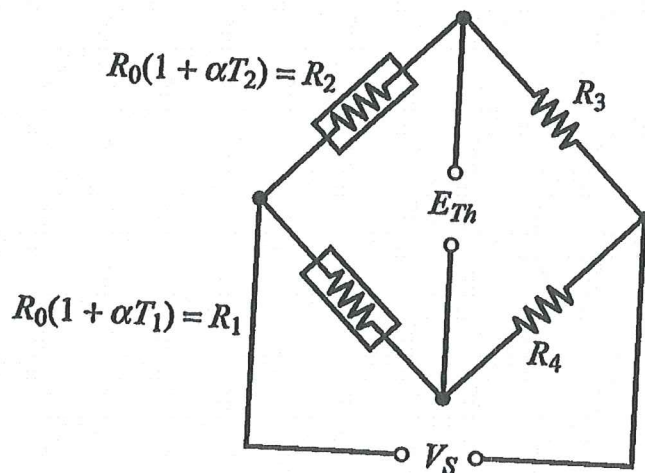


Figure 2: A two-element resistance sensor bridge.

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Answered (model)

content: 0,25 pnts, maximum score 10, max. pnts 11 (1 bonus)

(1) Algebra - theoretic (total 2 pnts)

- a) (0,5 pnt) b) (0,5 pnt) c) (0,5 pnt) d) (0,5 pnt)

(2) Pressure gauge (total 2 pnts)

memory unit ~~0,5~~ - 0,5
calculation error - 0,25
General Rule.

a) 1 pnt ; Φ

both modifying & interfering since slope & offset extrapolated at $P=0$ changes from 25°C to 35°C

b) ~~1 pnt~~ 1 pnt

$$O(I) = (K + K_n I_n) I + a + K_I I$$

I_n (mA) I_n (temp $^{\circ}\text{C}$) I (pressure p[ab]) I (temp. change $^{\circ}\text{C}$)

standard condition: $O(I) = K I + a$

($I_n = I_I = 0$)

from table:

$$\left[\begin{array}{l} K = 3,1 \frac{\text{mA}}{\text{b}} \\ a = 0,8 \text{ mA} \end{array} \right]$$

(= 3,9 - 3,1)

→ (0,5 pnt)

gem. schle:

$$(K + K_n I_n) = 3,7 \frac{\text{mA}}{\text{b}} \quad (1)$$

$$(a + K_E I_E) = -0,2 \text{ mA} \quad (2)$$

$$\Rightarrow (1) \Rightarrow 3,1 \frac{\text{mA}}{\text{b}} + K_n \cdot 10^\circ\text{C} = 3,7 \frac{\text{mA}}{\text{b}} \Rightarrow \boxed{K_n = 0,06 \frac{\text{mA}}{\text{C} \cdot \text{b}}}$$

$$(2) \Rightarrow 0,8 \text{ mA} + K_E \cdot 10^\circ\text{C} = -0,2 \text{ mA} \Rightarrow \boxed{K_E = -0,1 \frac{\text{mA}}{\text{C}}}$$

0,5 mA

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Question (3) (total 2 pnds)

$$(a) \quad V_{out} = (K + K_A I_{r2}) (E_{in} - K_F V_{out}) K_A$$

$$\Rightarrow V_{out} = \frac{(K + K_A I_{r2}) K_A}{1 + (K + K_A I_{r2}) K_F K_A} \cdot E_{in}$$

(0.5 pnd)

$$(b) \quad (K + K_A I_{r2}) K_F K_A \gg 1 \quad (\Rightarrow K_A \gg \frac{1}{K K_F})$$

$$\Rightarrow V_{out} = \frac{E_{in}}{K_F}$$

(0.5 pnd)

(c) high gain neg. feedback: (1 pnd)

* output not sensitive to interference effect.

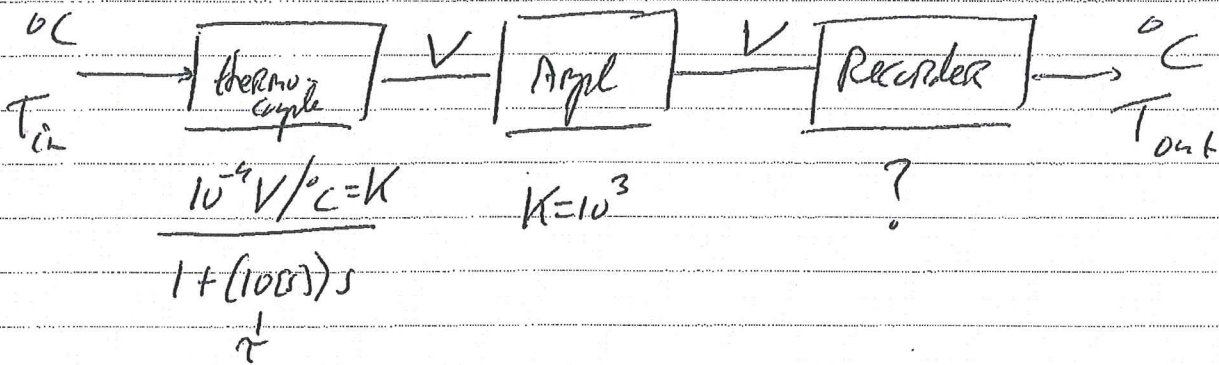
* Amplifier can be unstable, as long as amplification large.

* non-linearity in K, K_A, K_F eliminated as long as K_F part linear.

(Any of these answers gives points)

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(4) temp measurement system (2 pts)



$$G_{TC}(s) = \frac{10^{-4} \text{ V/}^\circ\text{C}}{1 + 10s}; \quad G_A(s) = 10^3; \quad G_R = \dots$$

$$a) G_{total} = \frac{1}{1 + (2s + 20s^2)} = \frac{1}{(1 + 10s)(1 + 2s)} = G_{TC} \times G_A \times G_R$$

$$\Rightarrow G_R(s) = \frac{10^{\circ\text{C/V}}}{(1 + 2s)} \quad \hookrightarrow \tau_R = 2 \text{ s}$$

0.5
pts

b) 7.0.7.

$$\frac{\Delta \tilde{T}_{out}}{\Delta \tilde{T}_{in}} = G(s) = \frac{1}{(1+10s)(1+2s)}$$

with $\Delta \tilde{T}_{in} = \frac{10^{-4}C}{s} \Rightarrow \Delta \tilde{T}_{out} = 10 \times \frac{1}{s(1+10s)(1+2s)}$
 (0,25 pV)

partial fraction:

$$\frac{1}{s(1+10s)(1+2s)} = \frac{A}{s} + \frac{B}{1+10s} + \frac{C}{1+2s}$$

$$A(1+10s)(1+2s) + Bs(1+2s) + Cs(1+10s) = 1$$

$$s^2 [20A + 2B + 10C] + s [12A + B + C] + A = 1$$

$\begin{matrix} = 0 & & = 0 & & \downarrow \\ & & & & A=1 \end{matrix}$

$$\Rightarrow \left. \begin{matrix} 12 + B + C = 0 \\ 20 + 2B + 10C = 0 \end{matrix} \right\} \begin{matrix} B + C = -12 \\ B + 5C = -10 \end{matrix}$$

$$-4C = -20$$

$$\hookrightarrow C = \frac{1}{2}$$

$$\hookrightarrow B + \frac{1}{2} = -12$$

$$B = -12,5 \quad \left(= -\frac{25}{2} \right)$$

$$\Rightarrow \Delta T_{\text{out}} = 10 \left[\frac{1}{s} - \frac{25/2}{1+10s} + \frac{1/2}{1+2s} \right] [^{\circ}\text{C}]$$

$$\left(\frac{25/20}{\frac{1}{10} + s} \right) \quad \left(\frac{1/4}{\frac{1}{2} + s} \right)$$

} \mathcal{L}^{-1}

$$\Delta T_{\text{out}}(t) = 10 \left[p_0(t) - \frac{25}{20} e^{-t/10} + \frac{1}{4} e^{-t/2} \right] [^{\circ}\text{C}]$$

(0,25 ph11)

c) z.o.z.

c) [^{0.5} ~~1~~ pwr total]

$$G(s) = \frac{1}{1+10s} \cdot \frac{1}{1+2s}$$

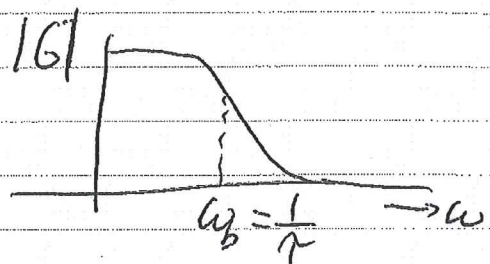
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$\tau_{tc} = 10 [s]$ $\tau_R = 2 [s]$

bandwidth defined:

$$|G(s=j\omega)| \stackrel{?}{\geq} \frac{1}{\sqrt{2}}$$

(0.707 pwr)



Since $\tau_{tc} \gg \tau_R \Rightarrow \omega_b = \frac{1}{\tau_{tc}} = 0,1 \text{ KHz}$

"exact solution"

$$|G|^2 = \frac{1}{(1+10j\omega)(1+2j\omega)} \cdot \frac{1}{(1-10j\omega)(1-2j\omega)}$$
$$= \frac{1}{[1+100\omega^2][1+4\omega^2]} \leq \frac{1}{2}$$

$\hookrightarrow \omega_b = 0,1 \text{ KHz}$

(0.5 pwr)

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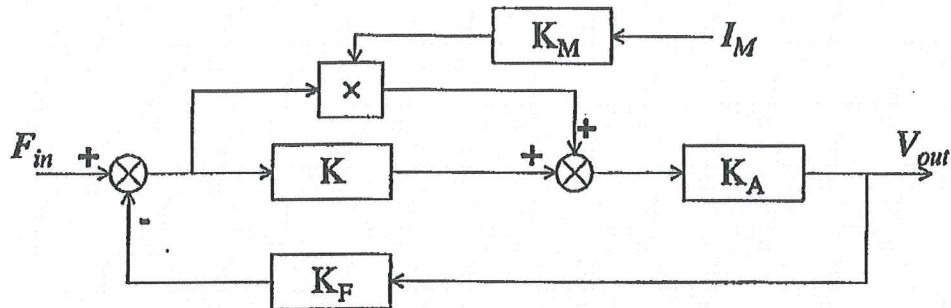


Figure 1: A high-gain negative feedback system.

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Question 5: Two-element resistance sensor bridge (2+1 points)

Consider a two-element resistance sensor bridge as indicated in Fig. 2. The bridge consists of two identical metal resistance sensors. One sensor is placed at a temperature T_1 (in °C) and the other placed at a fixed reference temperature $T_2=0$ °C. The formula for resistance as a function of temperature is given in the figure with the temperature coefficient $\alpha=5 \times 10^{-3}$ °C⁻¹ and $R_0=100$ Ω. The sensor indicated with T_1 operates in a range between 0 and 50 °C.

- What is the choice for R_3/R_4 such that $E_{Th}=0$ when $T_1=T_2$ (balanced bridge). Motivate your answer.
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- Take $V_S=12$ V and $R_3=R_4=10$ kΩ. How large is the maximum power dissipation through sensor R_1 ?

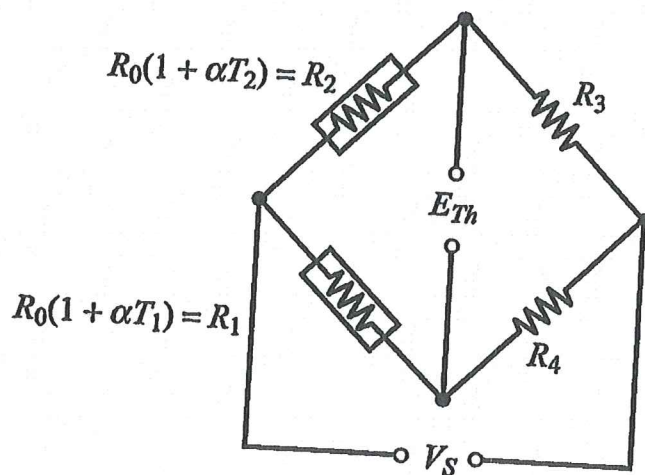


Figure 2: A two-element resistance sensor bridge.

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Answered (model)

each: 0,25 pnts, maximum score 10, max. pnts 11 (1 bonus)

(1) Algebra - theory (total 2 pnts)

- a) 0,5 pnt b) 0,5 pnt c) 0,5 pnt d) 0,5 pnt

(2) Pressure gauge (total 2 pnts)

Missing unit ~~0,5~~ general Rule.
evaluation error -0,25

a) 1 pnt ; Φ

both modifying & interfering since slope & offset
extrapolated at $P=0$ changes from 25°C to 35°C

b) ~~1 pnt~~ 1 pnt

$$O(I) = (K + K_n I_n) \underset{\substack{\text{temp } ^\circ\text{C} \\ \text{pressure} \\ \text{p [b]}}}{I} + a + K_I \underset{\substack{\text{temp. change} \\ ^\circ\text{C}}}{I}$$

I_n (nA)

standard condition: $O(I) = KI + a$

$(I_n = I_I = 0)$

from table:

$$\left[\begin{array}{l} K = 3,1 \frac{\text{nA}}{\text{b}} \\ a = 0,8 \text{ nA} \end{array} \right] \rightarrow 0,5 \text{ pnt} \quad (= 39 - 31)$$

freie Able:

$$(K + K_{12} I_{12}) = 3,7 \frac{\text{mA}}{\text{b}} \quad (1)$$

$$(a + K_{12} I_{12}) = -0,2 \text{ mA} \quad (2)$$

$$\Rightarrow (1) \Rightarrow 3,1 \frac{\text{mA}}{\text{b}} + K_{12} \cdot 10^\circ\text{C} = 3,7 \frac{\text{mA}}{\text{b}} \Rightarrow \boxed{K_{12} = 0,06 \frac{\text{mA}}{^\circ\text{C} \cdot \text{b}}}$$

$$(2) \Rightarrow 0,8 \text{ mA} + K_{12} \cdot 10^\circ\text{C} = -0,2 \text{ mA} \Rightarrow \boxed{K_{12} = -0,1 \frac{\text{mA}}{^\circ\text{C}}}$$

0,5 mA

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Question (3) (total 2 pnts)

$$(a) \quad V_{out} = (K + K_{in} I_{in}) (F_{in} - K_F V_{out}) K_A$$

$$\Rightarrow V_{out} = \frac{(K + K_{in} I_{in}) K_A}{1 + (K + K_{in} I_{in}) K_F K_A} F_{in}$$

(1.5 pnt)

$$(b) \quad (K + K_{in} I_{in}) K_F K_A \gg 1 \quad (\Rightarrow K_A \gg \frac{1}{K K_F})$$

$$\Rightarrow V_{out} = \frac{F_{in}}{K_F}$$

(0.5 pnt)

(c) high gain neg. feedback: (1 pnt)

* output not sensitive to interference effect.

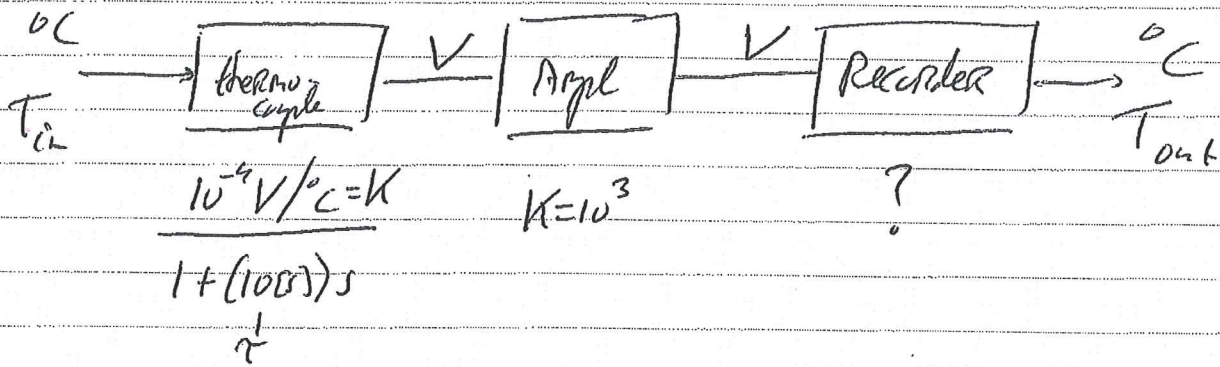
* Amplifier can be unstable, as long as β amplification large.

* non-linearity in K, K_{in}, K_A eliminated as long as K_F part linear.

(Any of these answers gives points)

POMIS exam 23/01/2014

(1) temp. measurement system (2 points)



$$G_{TC}(s) = \frac{10^{-4} \text{ V/}^\circ\text{C}}{1 + 10s}; \quad G_A(s) = 10^3; \quad G_R = \dots$$

$$a) \quad G_{\text{total}} = \frac{1}{1 + 25 + 20s^2} = \frac{1}{(1 + 10s)(1 + 2s)} = G_{TC} \times G_A \times G_R$$

$$\Rightarrow G_R(s) = \frac{10^0 \text{ }^\circ\text{C/V}}{(1 + 2s)}$$

$L \tau_R = 2 \text{ s}$

0.5
pts

b) 7.0.7.

$$\frac{\tilde{\Delta T}_{out}}{\tilde{\Delta T}_{in}} = G(s) = \frac{1}{(1+10s)(1+2s)}$$

$$\text{with } \tilde{\Delta T}_{in} = \frac{10^{-4} \text{V}}{s} \Rightarrow \tilde{\Delta T}_{out} = 10 \times \frac{1}{s(1+10s)(1+2s)}$$

(0,25 pV)

partial fraction:

$$\frac{1}{s(1+10s)(1+2s)} = \frac{A}{s} + \frac{B}{1+10s} + \frac{C}{1+2s}$$

$$A(1+10s)(1+2s) + Bs(1+2s) + Cs(1+10s) = 1$$

$$s^2 [20A + 2B + 10C] + s [12A + B + C] + A = 1$$

=0

=0

$$A = 1$$

$$\Rightarrow 12 + B + C = 0$$

$$20 + 2B + 10C = 0$$

$$B + C = -12$$

$$B + 5C = -10$$

$$-4C = -20$$

$$C = \frac{1}{2}$$

$$\hookrightarrow B + \frac{1}{2} = -12$$

$$B = -12,5 \quad \left(= -\frac{25}{2} \right)$$

(0,25 pV)

$$\Rightarrow \Delta T_{\text{out}} = 10 \left[\frac{1}{s} - \frac{25/2}{1+10s} + \frac{1/2}{1+2s} \right] [^{\circ}\text{C}]$$

$$\left(\frac{25/20}{\frac{1}{10} + s} \right) \quad \left(\frac{1/4}{\frac{1}{2} + s} \right)$$

} \mathcal{L}^{-1}

$$\Delta T_{\text{out}}(t) = 10 \left[p(t) - \frac{25}{20} e^{-t/10} + \frac{1}{4} e^{-t/2} \right] [^{\circ}\text{C}]$$

(0,25 p411)

c) z.o.z.

c) [~~1~~^{0.5} pms total]

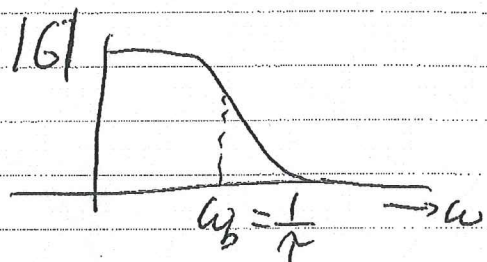
$$G(s) = \frac{1}{1+10s} \cdot \frac{1}{1+2s}$$

\swarrow \searrow
 $\tau_{tc} = 10 [s]$ $\tau_R = 2 [s]$

bandwidth defined:

$$|G(s=j\omega)| \stackrel{!}{\geq} \frac{1}{\sqrt{2}}$$

(0.707 pms)



Since $\tau_{tc} \gg \tau_R \Rightarrow \omega_b = \frac{1}{\tau_{tc}} = 0,1 \text{ kHz}$

"exact solution"

$$|G|^2 = \frac{1}{(1+10j\omega)(1+2j\omega)} \cdot \frac{1}{(1-10j\omega)(1-2j\omega)}$$
$$= \frac{1}{[1+100\omega^2][1+4\omega^2]} \leq \frac{1}{2}$$

$\hookrightarrow \omega_b \approx 0,1 \text{ kHz}$

(0.707 pms)

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(5) (total: 3 pnts (1 bonus))

a)
$$E_{th} = \frac{R_1 R_3 - R_2 R_4}{(R_1 + R_4)(R_2 + R_3)} V_s$$
 (Wheatstone bridge)

$T_1 = T_2 (= 0^\circ C) \Rightarrow R_1 = R_2$, hence $R_3 = R_4$ che-

(0.5 pnts)

$E_{th} = 0$
for that
condition.

b)
$$E_{th} = \left(\frac{1}{1 + \frac{R_3}{R_1}} - \frac{1}{1 + \frac{R_3}{R_2}} \right) V_s$$

$$= \left(\frac{1}{1 + \frac{l\alpha}{l\alpha(1+\alpha T_1)}} - \frac{1}{1 + \frac{l\alpha}{l\alpha}} \right) V_s$$

$$= \left(\frac{(1+\alpha T_1)}{(1+\alpha T_1)+1} - \frac{1}{2} \right) V_s = \left(\frac{(1+\alpha T_1)}{(2+\alpha T_1)} - \frac{1}{2} \right) V_s$$

$T_1 = 0 \Rightarrow E_{th} = 0$ (balanced) 0,25

$$T_1 = 50^\circ C \Rightarrow E_{th} = \left(\frac{(1+\alpha T_1)}{(2+\alpha T_1)} - \frac{1}{2} \right) V_s \equiv 1 V$$

$\hookrightarrow [V_s = 10 V]$

(0.5 pnts)

@ $T = 25^\circ\text{C}$

$$\Rightarrow E_{th} = \left(\frac{(1 + \frac{1}{8})}{(2 + \frac{1}{8})} - \frac{1}{2} \right) 1.5\text{V} = 0.53\text{ [V]}$$

for linear response: $E_{th}^{\text{ideal}} = 0.5\text{V}$

hence non-linearity: $\frac{(0.53 - 0.5)}{0.5} \times 100\% = \frac{0.03}{0.5} \times 100\% = 6\%$

(full span) 0.5 (mV)

c)
$$E_{th} = \left[\frac{1}{1 + \frac{R_4 (= R_3)}{R_0 (1 + \alpha T_1)}} - \frac{1}{1 + \frac{R_3 (= R_4)}{R_0 (1 + \alpha T_2)}} \right] V_s$$

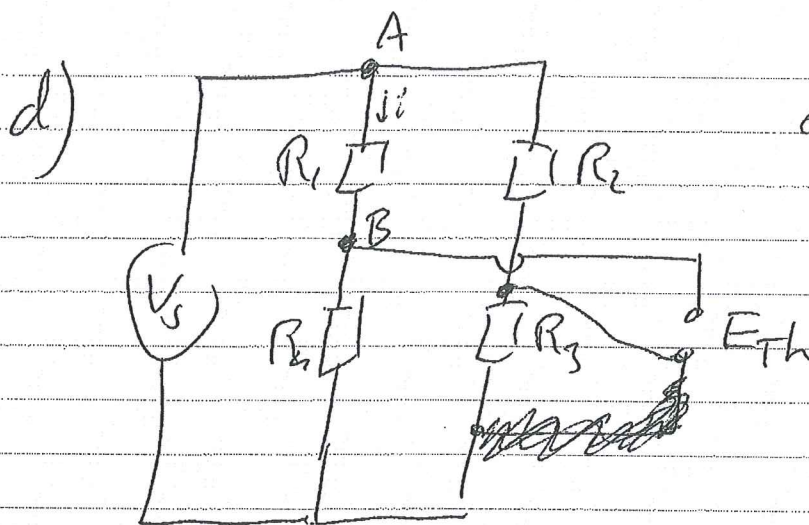
$\gg 1$ $\gg 1$

$$\approx \left[\frac{(1 + \alpha T_1)}{R_3/R_0} - \frac{(1 + \alpha T_2)}{R_3/R_0} \right] V_s$$

$$= V_s \left(\frac{R_0}{R_3} \right) \alpha [T_1 - T_2] = 0.5 \text{ (mV)}$$

So, take $R_3 = R_4 \gg R_0$

Price: sensitivity is very small since R_0/R_3 becomes small



$$i = \frac{V_s}{R_1 + R_2}$$

$$\begin{aligned} \hookrightarrow V_{AB} &= i R_1 \\ &= V_s \left[\frac{R_1}{R_1 + R_2} \right] \end{aligned}$$

$$V_{AB} = V_A - V_B = V_s - V_s \frac{R_2}{R_1 + R_2} = V_s \left(\frac{R_1}{R_1 + R_2} \right)$$

power through R_1 : $P = \frac{V_{AB}^2}{R_1} = V_s^2 \frac{R_1}{(R_1 + R_2)^2} = V_s^2 \frac{R_1}{R_2^2}$

maximum when R_1 maximum, e.g. $T = 50^\circ\text{C}$

$$P = (12)^2 \left[\frac{100 \times (1 + 0,2x)}{(10 \cdot 10^3)^2} \right] \text{ [W]}$$

$$= 144 \cdot \left[\frac{125}{10^8} \right] = 1,8 \cdot 10^{-4} \text{ W}$$

$$= \underline{\underline{0,18 \text{ mW}}}$$

0,5 pW

0,5 pW

